Selected Solutions for Chapter 12: Binary Search Trees

Solution to Exercise 12.1-2

In a heap, a node's key is $\geq$ both of its children's keys. In a binary search tree, a node's key is $\geq$ its left child's key, but $\leq$ its right child's key.

The heap property, unlike the binary-searth-tree property, doesn't help print the nodes in sorted order because it doesn't tell which subtree of a node contains the element to print before that node. In a heap, the largest element smaller than the node could be in either subtree.

Note that if the heap property could be used to print the keys in sorted order in $O(n)$ time, we would have an $O(n)$-time algorithm for sorting, because building the heap takes only $O(n)$ time. But we know (Chapter 8) that a comparison sort must take $\Omega(n \lg n)$ time.

Solution to Exercise 12.2-7

Note that a call to TREE-MINIMUM followed by $n-1$ calls to TREE-SUCCESSOR performs exactly the same inorder walk of the tree as does the procedure INORDER-TREE-WALK. INORDER-TREE-WALK prints the TREE-MINIMUM first, and by definition, the TREE-SUCCESSOR of a node is the next node in the sorted order determined by an inorder tree walk.

This algorithm runs in $\Theta(n)$ time because:

- It requires $\Omega(n)$ time to do the n procedure calls.
- It traverses each of the $n-1$ tree edges at most twice, which takes $O(n)$ time.

To see that each edge is traversed at most twice (once going down the tree and once going up), consider the edge between any node u and either of its children, node v. By starting at the root, we must traverse (u, v) downward from u to v, before traversing it upward from v to u. The only time the tree is traversed downward is in code of TREE-MINIMUM, and the only time the tree is traversed upward is in code of TREE-SUCCESSOR when we look for the successor of a node that has no right subtree.

Suppose that v is u's left child.

- Before printing u, we must print all the nodes in its left subtree, which is rooted at v, guaranteeing the downward traversal of edge (u, v).
- After all nodes in u's left subtree are printed, u must be printed next. Procedure TREE-SUCCESSOR traverses an upward path to u from the maximum element (which has no right subtree) in the subtree rooted at v. This path clearly includes edge (u, v), and since all nodes in u's left subtree are printed, edge (u, v) is never traversed again.

Now suppose that v is u's right child.

- After u is printed, TREE-SUCCESSOR(u) is called. To get to the minimum element in u's right subtree (whose root is v), the edge (u, v) must be traversed downward.
- After all values in u's right subtree are printed, TREE-SUCCESSOR is called on the maximum element (again, which has no right subtree) in the subtree rooted at v. TREE-SUCCESSOR traverses a path up the tree to an element after u, since u was already printed. Edge (u, v) must be traversed upward on this path, and since all nodes in u's right subtree have been printed, edge (u, v) is never traversed again.

Hence, no edge is traversed twice in the same direction.

Therefore, this algorithm runs in $\Theta(n)$ time.

Solution to Exercise 12.3-3

Here's the algorithm:

TREE-SORT(A)
let T be an empty binary search tree
for $i = 1$ **to** n
    TREE-INSERT(T, $A[i]$)
INORDER-TREE-WALK($T.root$)

Worst case: $\Theta(n^2)$—occurs when a linear chain of nodes results from the repeated TREE-INSERT operations.

Best case: $\Theta(n \lg n)$—occurs when a binary tree of height $\Theta(\lg n)$ results from the repeated TREE-INSERT operations.

Solution to Problem 12-2

To sort the strings of S, we first insert them into a radix tree, and then use a preorder tree walk to extract them in lexicographically sorted order. The tree walk outputs strings only for nodes that indicate the existence of a string (i.e., those that are lightly shaded in Figure 12.5 of the text).

Correctness

The preorder ordering is the correct order because:

- Any node's string is a prefix of all its descendants' strings and hence belongs before them in the sorted order (rule 2).
- A node's left descendants belong before its right descendants because the corresponding strings are identical up to that parent node, and in the next position the left subtree's strings have 0 whereas the right subtree's strings have 1 (rule 1).

Time

$\Theta(n)$.

- Insertion takes $\Theta(n)$ time, since the insertion of each string takes time proportional to its length (traversing a path through the tree whose length is the length of the string), and the sum of all the string lengths is n.
- The preorder tree walk takes $O(n)$ time. It is just like INORDER-TREE-WALK (it prints the current node and calls itself recursively on the left and right subtrees), so it takes time proportional to the number of nodes in the tree. The number of nodes is at most 1 plus the sum (n) of the lengths of the binary strings in the tree, because a length-i string corresponds to a path through the root and i other nodes, but a single node may be shared among many string paths.